Name: Notes

SM275 · Mathematical Methods for Economics Fall 2019 · Uhan

Exam 1 – 23 September 2019

Instructions

- You have until the end of the class period to complete this exam.
- You may use your calculator.
- You may consult the SM275 Formula Table given to you with this exam.
- You may not consult any other outside materials (e.g. notes, textbooks, homework, computer).
- **Show all your work.** To receive full credit, your solution must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.

Problem	Weight	Score
1	1	
2	1	
3	2	
4	1	
5	2	
6	2	
7	2	
8	1	
9	2	
10	2	
11	2	
12	2	
13	2	
14	2	
Total		/ 240

For Problems 1-4, consider the following setting.

Suppose we win the lottery. We are given 30 annual payments of $100 each, with the first payment given now. Assume that whenever we get a payment, we put it in a savings account earning interest at an annual rate of 0.02, compounded annually.

Let A_n be the amount in the savings account after n years.

Problem 1. Write the DS for this setting.

- Many of you struggled with this problem.
- Remember that the DS is "the setup": an equation that relates the value of A_n with the values of $A_{n-1}, A_{n-2}, \dots$
- See Examples 1 and 2 from Lesson 4 for similar examples.

Problem 2. Write the IC for this setting.

- Most of you answered this problem correctly.
- Again, see Examples 1 and 2 from Lesson 4 for relevant examples.

Problem 3. Find the particular solution that satisfies the IC.

- You did not need to memorize the formula for the particular solution to a first-order linear DS to do this problem.
- Remember the universal method for finding a particular solution that we discussed during our concept review:
  1. Find the general solution of the DS.
  2. Plug the IC into the general solution, solve for the free constants.
  3. Plug the free constants back into the general solution to get the particular solution.

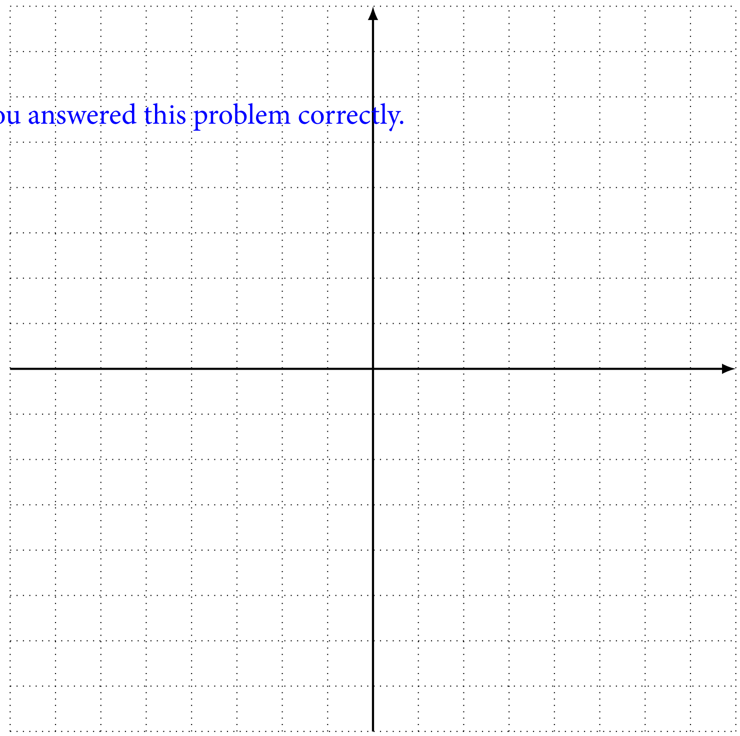
Problem 4. Use the particular solution you found in Problem 3 to find the amount in the account after 30 payments.

- Be careful: if the first payment occurs at time $n = 0$, when does the 30th payment occur?

For Problems 5-7, consider the DS

$$A_{n+1} = 2A_n + 1 \qquad n = 0, 1, 2, \ldots$$

Problem 5. Draw the cobwebs with $A_0 = -3$ and $A_0 = 1$. Don't forget to indicate the direction of the cobwebs.



- Almost all of you answered this problem correctly.

Problem 6. Explain why the fixed point of the DS is -1.

- Almost all of you answered this problem correctly.
- Be careful with your algebra and arithmetic!

Problem 7. Is the fixed point -1 attracting, repelling, or neither? Briefly explain.

- Almost all of you answered this problem correctly.

For Problems 8 and 9, consider the discrete market model

$$D_t = S_t \quad (1)$$
$$D_t = 18 - P_t \quad (2)$$
$$S_t = -2 + 3P_{t-1} \quad (3)$$

where at time t, D_t is the demand, S_t is the supply, and P_t is the price. In addition, suppose $P_0 = 8$.

Problem 8. In words, briefly explain why equation (2) makes sense from an economic perspective.

- Most of you answered this problem correctly.
- Some of you simply stated that equation (2) shows that demand is related to price. Be more specific: how is demand related to price?

Problem 9. Using the equations above, find a first order linear DS that describes how the price evolves over time. Your answer should look like: "$P_{t+1} = \ldots$" Do not solve the DS.

- Many of you struggled with this problem.
- Take a look at Section 2 of Lesson 5. Apply the same ideas here, except use the specific numbers given in equations (1)-(3) above. (Do not simply take the DS we obtained in Lesson 5 and plug in numbers. Show how to derive the DS.)

For Problems 10 and 11, consider the DS

$$A_{n+2} = A_{n+1} + 2A_n + 6 \qquad n = 0, 1, 2, \ldots$$

Problem 10. Find the general solution.

- Most of you had the right idea here.
- This type of problem requires you to be precise over the course of many steps. Take care when identifying a, b and c, finding the roots of r and s, and selecting the appropriate general solution.

Problem 11. Find the particular solution satisfying the IC $A_0 = 1$, $A_1 = 2$.

- Be careful when plugging in the IC into the general solution. In particular, make sure you're using the correct values of n.

Problem 12. Consider following the national income model, with marginal propensity to consume $m = \frac{3}{4}$ and accelerator $\ell = \frac{1}{3}$:

$$\left.\begin{aligned} T_n &= C_n + I_n + G_n \\ C_{n+1} &= \frac{3}{4} T_n \\ I_{n+1} &= \frac{1}{3}(C_{n+1} - C_n) \\ G_n &= 1 \end{aligned}\right. \qquad n = 0, 1, 2, \dots$$

where at time n, T_n is the total national income, C_n is the amount of consumer expenditures, I_n is the amount of private investment, and G_n is the amount of government expenditures. We showed that we can rewrite this model as the following DS:

$$T_{n+2} = T_{n+1} - \frac{1}{4} T_n + 1 \qquad n = 0, 1, 2, \dots$$

Suppose $C_0 = 2$ and $I_0 = 1$. Find the IC for the DS.

- See Example 1c in Lesson 7 for a similar example.
- Remember the definition of the IC for a second-order DS (Lesson 6).

For Problems 13 and 14, consider the following DS:

$$A_{n+2} = -\frac{2}{3}A_{n+1} + \frac{1}{3}A_n + 8 \qquad n = 0, 1, 2, \ldots$$

The general solution to this DS is

$$A_n = c_1(-1)^n + c_2\left(\frac{1}{3}\right)^n + 6.$$

Problem 13. Find the fixed point of this DS.

- Almost all of you answered this problem correctly.
- Be careful with your algebra and arithmetic!

Problem 14. Is the system stable, unstable, or neither? Briefly explain.

- Most of you correctly noted that $(-1)^n$ oscillates between -1 and 1 as $n \to \infty$. So, under what conditions does A_n oscillate?
- Some of you said that the system is stable when $c_1 = 0$ because in this case, A_n converges to a finite value as $n \to \infty$. This is incorrect! For a system to be stable, A_n must converge to a finite value as $n \to \infty$ for all initial conditions, or equivalently, all values of c_1 and c_2.

Additional page for scratchwork or solutions